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A METHOD FOR THE RAPID ESTIMATION OF TURBULENT BOUNDARY-LAYER THICKNESSES FOR CALCULATING PROFILE DRAG

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ADVANCE CONFIDENTIAL REPORT

A METHOD FOR THE RAPID ESTIMATION OF TURBULENT BOUNDARY-LAYER THICKNESSES FOR CALCULATING PROFILE DRAG

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SUMMARY

An analysis is developed that makes it possible to integrate von Kármán's boundary-layer momentum equation directly. For this purpose the skin-friction coefficient is expressed as a function of the Reynolds number based on the boundary-layer momentum thickness and the boundary-layer shape is assumed to be constant. The integrated equation permits the boundary-layer momentum thickness at the airfoil trailing edge to be rapidly determined for the estimation of airfoil profile-drag coefficients.

INTRODUCTION

Recent increases in size and speed of modern aircraft have emphasized the need for a method of estimating rapidly the profile drag of wings. One of the most time-consuming operations in a profile-drag estimate has been the calculation of the thickness of the turbulent boundary layer. Squire and Young (reference 1) calculate the thickness of the turbulent boundary layer by a step-by-step integration of von Kármán's boundary-layer momentum equation (reference 2). A logarithmic skin-friction formula designed to fit Schlichting's approximate formula for the skin friction (reference 3) is used in the integration. Reference 4 gives a method for the determination of the turbulent boundary-layer thickness which eliminates the step-by-step computations of reference 1 but requires that suitable approximations to the airfoil pressure distribution be made. The method of reference 5 eliminates ~~both~~ the step-by-step computations of reference 1 and the necessity for making approximations to the airfoil pressure distribution required in reference 4;

however, reference 5 is restricted to a particular boundary-layer shape and does not explicitly indicate how to use an arbitrary skin-friction law with the integrated von Kármán equation.

In the present paper the step-by-step computation of reference 1 is eliminated along with the necessity for making approximations to the airfoil pressure distributions, which were required in reference 4. A more general specification of the boundary-layer shape than that given in reference 5 is used and a method for use of any skin-friction law with the integrated von Kármán equation is presented. The integrated form of the von Kármán momentum equation appears applicable either to the laminar or the turbulent boundary layer provided that the appropriate and constant shape of the boundary layer is assumed and the appropriate formula for the skin friction is used. For the turbulent boundary layer, a skin-friction formula based directly on the data obtained from experiments in pipes (reference 2) is presented in a form readily usable for boundary-layer calculations.

The present work provides a method for the calculation of the thickness of the turbulent boundary layer which is more rapid than that of reference 1, more flexible than that of reference 4, and more general than that of reference 5. The flexibility of the method arises from the fact that the effects on the calculated turbulent boundary-layer thickness of local changes in the airfoil pressure distribution, changes in the boundary-layer shape, and changes in the skin-friction formulas can all be easily evaluated. The method, however, like those of references 1, 4, and 5 cannot predict the onset of turbulent separation and, if the danger of turbulent separation is thought to exist, the method of reference 6 may be used to check for such a condition. Although the present method may be used in such cases to obtain momentum thickness up to the point of separation, it should not be used in the determination of the drag coefficients. The present method can be used to calculate momentum thicknesses for flow over rough as well as smooth surfaces, if a curve of skin-friction coefficient for the rough surface is available.

The skin-friction equations presented should provide a good estimate of the turbulent boundary-layer thickness for use in profile-drag estimates when the airfoil is smooth and there is no danger of turbulent separation.

SYMBOLS

c	airfoil chord
c_{d_o}	section profile-drag coefficient
H	boundary-layer shape parameter for which values are given in reference 6 $\left(\frac{\delta^*}{\theta}\right)$
k	constant in equation for skin-friction coefficient
n	constant in equation for skin-friction coefficient
q	dynamic pressure, pounds per square foot $\left(\frac{\rho}{2}U^2\right)$
R_c	wing Reynolds number based on chord and free-stream velocity
R_θ	boundary-layer Reynolds number $\left(\frac{U\theta}{\nu}\right)$
s	distance along airfoil surface, usually taken equal to x
s_o	position on surface at start of integration
u	local velocity inside boundary layer
U	local velocity outside boundary layer
U_o	free-stream velocity
U_1	velocity at s_o/c
x	distance along airfoil chord
y	perpendicular distance from wing surface
δ	full boundary-layer thickness
δ^*	displacement thickness $\left[\int_o^\delta \left(1-\frac{u}{U}\right)dy\right]$
ρ	density
θ	momentum thickness $\left[\int_o^\delta \frac{u}{U}\left(1-\frac{u}{U}\right)dy\right]$
θ_o	momentum thickness at s_o/c
τ_o	skin friction per unit area, pounds per square foot
ν	kinematic viscosity

The subscript t is used to indicate the condition at the trailing edge.

[redacted]

ANALYSIS

The von Karman boundary-layer momentum equation, which is used to calculate the momentum thickness of the boundary layer, is given in reference 2 and may be written as

$$\frac{d\theta}{ds} + \frac{H + 2}{2} \frac{\theta}{q} \frac{dq}{ds} = \frac{\tau_o}{2q}$$

The momentum equation is a differential equation that can be integrated if the boundary-layer shape, and therefore H, is assumed to be constant and if the local skin-friction coefficient $\frac{\tau_o}{2q}$ is expressible as a power function of R_θ.

In order to integrate the equation the skin-friction coefficient is written as

$$\frac{\tau_o}{2q} = \frac{k}{{R_\theta}^n} = \frac{k\nu^n}{\theta^n U^n}$$

where k and n are constants that are to be determined. The momentum equation is written as

$$\frac{d\theta}{ds} + \frac{H + 2}{U} \frac{dU}{ds} \theta = \frac{k\nu^n}{U^n} \theta^{-n}$$

which is a differential equation of the Bernoulli type. After the equation is integrated and the arbitrary constant arising from the integration is evaluated, the result is

$$\left(\frac{\theta}{c}\right)_{s/c} = \frac{1}{\left(\frac{U}{U_o}\right)_{s/c}^{H+2}} \left[\frac{(1+n)k}{{R_c}^n} \int_{s_o/c}^{s/c} \left(\frac{U}{U_o}\right)^{(H+1)(n+1)+1} d\frac{s}{c} + \left(\frac{\theta_o}{c}\right)^{n+1} \left(\frac{U_1}{U_o}\right)^{(H+2)(n+1)}\right]^{\frac{1}{1+n}} \tag{1}$$

The definite integral occurring in equation (1) may be evaluated by either an analytical or a graphical method, whichever is more convenient.

DISCUSSION

When the momentum equation was integrated, the assumption was made that the shape of the boundary layer, and therefore H, was constant over the chord. Although the assumption is not true in general, the error in θ will usually be small because of the manner in which H appears in the momentum equation.

Experience has indicated that, for the calculation of the thickness of the turbulent boundary layer, the value of H may be chosen close to 1.4 for airfoils that have small pressure recoveries but, for airfoils that have high pressure recoveries, which are thick or operating at high lift coefficients, H should be chosen closer to 1.6. In references 1, 4, and 5, H was chosen as constant and equal to 1.4. The thickness of the laminar boundary layer can usually be obtained with sufficient accuracy for profile-drag calculations if the integrated momentum equation is used, together with a value of H of 2.592, which is the value for the Blasius flat-plate profile (reference 2).

In order to use the present form of the integrated momentum equation, it is necessary that the skin-friction coefficient be written as

$$\frac{\tau_o}{2q} = \frac{k}{{R_\theta}^n}$$

For the laminar boundary layer having the Blasius shape, $n = 1$ and $k = 0.2205$.

A skin-friction formula for the turbulent boundary layer which has the required form is that of Falkner (reference 7) who, after analyzing various data obtained from flat plates, suggested the relation

$$\frac{\tau_o}{2q} = \frac{0.006535}{{R_\theta}^{1/6}}$$

[redacted]

A skin-friction formula based directly on pipe experiments can be derived by use of equations (22.21), (22.18), and (22.17) of reference 2, pages 142 and 143. This formula

$$\frac{\tau_o}{2q} = \frac{1}{\left[2.5 \log_e \frac{R_\theta}{2.5\left(1 - 5\sqrt{\tau_o/2q}\right)} + 5.5\right]^2} \qquad (2)$$

is plotted in figure 1. The curve of equation (2) and the relation suggested by Falkner (reference 7) are plotted in figure 2. At values of R_θ above about 2000, the curves are found to be in good agreement but at values below 2000 diverge considerably.

The skin-friction formula derived in reference 3 indicates skin-friction coefficients at small values of R_θ that are in good agreement with equation (2). The skin-friction coefficients given by Falkner's equation are therefore probably too small at small values of R_θ.

Equation (2) is based directly on the pipe experiments to which the formula of Squire and Young (reference 1), although not derived in a direct manner, also owes its origin. This equation and a convenient plot of the equation are given in reference 6. Consideration of flat-plate skin-friction data and approximations, which were made in the interest of simplicity of expression, account for the difference between the Squire and Young formula and equation (2). The skin-friction coefficients indicated by use of the Squire and Young formula are slightly lower than those obtained by use of equation (2) at large values of R_θ but are about the same as those obtained by equation (2) at small values of R_θ. The introduction of equation (2) also serves to indicate how an arbitrary skin-friction formula may be used with equation (1).

In order to permit the integration of the momentum equation, the skin-friction relation chosen for use is approximated by a simple power function of R_θ. This approximation is made by using two values of R_θ and the skin-friction coefficients associated with the two R_θ values to evaluate the two constants, n and k, in the equation

$$\frac{\tau_o}{2q} = \frac{k}{{R_\theta}^n}$$

The formulas to be used are

$$n = \frac{\log_e \frac{(\tau_o/2q)_1}{(\tau_o/2q)_2}}{\log_e \frac{R_{\theta_2}}{R_{\theta_1}}}$$

$$k = \left(\frac{\tau_o}{2q}\right)_1 {R_{\theta_1}}^n$$

where the subscript 1 identifies the skin-friction coefficient occurring at R_{θ_1}, the smaller value of R_θ, and the subscript 2 denotes the skin-friction coefficient occurring at R_{θ_2}, the larger value of R_θ. Experience has indicated that, if R_{θ_2} is chosen about ten times as large as R_{θ_1}, the approximation to equation (2) will be good over the entire range of R_θ values of the turbulent boundary layer likely to be encountered in a specific case. The beginning of the turbulent boundary layer has been found to be the region in which it is most important to have a good approximation of the skin-friction curve. Because of this fact, R_{θ_1} should be chosen close to the initial value of R_θ of the turbulent boundary layer, which is assumed equal to the final value of R_θ in the laminar boundary layer.

After the momentum thickness of the boundary layer at the trailing edge has been obtained, the profile-drag coefficient is calculated by the relation developed by Squire and Young in reference 1

$$c_{d_o} = 2\left(\frac{\theta}{c}\right)_t \left[\left(\frac{U}{U_o}\right)_t\right]^{\frac{H+5}{2}}$$

The subscript t is used to identify the condition at the trailing edge.

COMPARISON WITH EXPERIMENT

Calculations of profile-drag coefficients have been made by the method described herein and the results have been compared with those obtained by other investigators. The profile-drag coefficient of the 25-percent-thick airfoil at a lift coefficient of 0.25 and a Reynolds number of 8.2×10^6, for which data are given in reference 1, was calculated and compared with the experimental value of 0.0080 given in reference 1. When the equation

$$\frac{\tau_o}{2q} = \frac{0.006535}{{R_\theta}^{1/6}}$$

was used, the calculated profile-drag coefficient was 0.0076. When the equation

$$\frac{\tau_o}{2q} = \frac{0.00905}{{R_\theta}^{0.2028}}$$

was used to fit equation (2) for values of R_θ between 1,100 and 11,000, the calculated drag coefficient was 0.0077. Squire and Young in reference 1 obtained a calculated value of 0.0079. In all three cases the value of H was taken equal to 1.4.

Calculations of the boundary-layer momentum thicknesses along the chord and the profile-drag coefficients of the NACA 0012 airfoil at zero angle of attack for four Reynolds numbers have been made and compared with the experimental results obtained from reference 8. The comparison is shown in the following table:

R_c	c_{d_0}			
	Experimental (reference 8)	Calculated, Squire and Young (reference 8)	Calculated, Falkner's equation	Calculated, equation (2)
2.675×10^6	0.0071	0.0074	0.0067	0.0069
3.780	.0070	.0072	.0069	.0070
5.350	.0068	.0071	.0069	.0070
7.560	.0067	.0071	.0069	.0069

[redacted]

The profile-drag coefficients that were calculated by using Falkner's formula in one calculation and the relation

$$\frac{\tau_o}{2q} = \frac{0.00934}{R_\theta^{0.2068}}$$

as an approximation to equation (2) between R_θ values of 900 and 9000 in the second calculation are given in this table, together with the experimental values from reference 8 and profile-drag coefficients calculated by the Squire and Young method in reference 8. The boundary-layer momentum thicknesses over the surface of the NACA 0012 airfoil, which were calculated by using the approximation to equation (2) and the formula suggested by Falkner, are given in figure 3 together with the experimental values of momentum thickness obtained from reference 8. The differences between the two calculated momentum thicknesses are caused by the use of the two different skin-friction formulas, the value of H being taken as 1.5 for both calculations.

In order to indicate the effect of the choice of H for the turbulent boundary layer on the calculated profile-drag coefficient of a thin airfoil at a small lift coefficient, the profile-drag coefficient of the NACA 0012 airfoil at zero angle of attack and a Reynolds number of 7.560×10^6 was also calculated for values of H of 1.4 and 1.6. The profile-drag coefficients were 0.0068 for a value of H of 1.4 and 0.0070 for a value of H of 1.6. The change in H caused a relatively small change in the profile-drag coefficient of the NACA 0012 airfoil at zero angle of attack because for this airfoil at zero angle of attack the change in velocity over the airfoil surface is small. As can be seen from equation (1), a change in the value of H will have greater effect on the calculated profile-drag coefficient for cases in which large changes in velocity occur (for example, on thick airfoils or on airfoils at large lift coefficients) than for cases in which the change in velocity is small.

SAMPLE CALCULATION

A calculation is given for the profile-drag coefficient of the NACA 0012 airfoil at a Reynolds number of 2.675×10^6 and zero angle of attack. The velocity distribution, which was obtained from reference 8, is given in figure 4 and is the experimental velocity distribution, uncorrected for tunnel-wall effects. At the Reynolds number of 2.675×10^6 the transition point was at $\frac{s}{c} = 0.48$.

The equation that was used for the calculation of the momentum thickness of the laminar boundary layer is obtained from equation (1) by placing $\theta_o/c = 0$, $n = 1$, $k = 0.2205$, and $H = 2.592$. The resulting equation is

$$\left(\frac{\theta}{c}\right)_{s/c} = \frac{0.664}{\left[\left(\frac{U}{U_o}\right)_{s/c}\right]^{4.592}} \frac{1}{R_c^{1/2}} \left[\int_0^{s/c} \left(\frac{U}{U_o}\right)^{8.184} d\frac{s}{c}\right]^{\frac{1}{2}}$$

or, for the particular case,

$$\left(\frac{\theta}{c}\right)_{\frac{s}{c}=0.48} = \frac{0.664}{(1.144)^{4.592}} \frac{1}{\sqrt{2.675 \times 10^6}} \left[\int_0^{0.48} \left(\frac{U}{U_o}\right)^{8.184} d\frac{s}{c}\right]^{\frac{1}{2}}$$

The value of $\int_0^{0.48} \left(\frac{U}{U_o}\right)^{8.184} d\frac{s}{c}$ is equal to 1.760.

Then

$$\left(\frac{\theta}{c}\right)_{\frac{s}{c}=0.48} = 0.0002901$$

The final value of R_θ in the laminar boundary, which is assumed to be the initial value of R_θ of the turbulent boundary layer, is then calculated:

$$R_\theta = R_c\left(\frac{\theta}{c}\,\frac{U}{U_o}\right)_{\frac{s}{c}=0.48}$$

$$= 2.675 \times 10^6 \times 0.0002901 \times 1.144$$

$$= 888$$

A value of 900 was chosen for R_{θ_1} and 9000 for R_{θ_2} for the approximation of equation (2) by a power function of R_θ. The computations for n and k are as follows: From figure (1)

$$R_{\theta_1} = 900$$

$$\frac{\tau_o}{2q} = 0.002289$$

$$R_{\theta_2} = 9000$$

$$\frac{\tau_o}{2q} = 0.001421$$

$$n = \frac{\log\frac{0.002289}{0.001421}}{\log 10} = \frac{0.476}{2.302}$$

$$= 0.2068$$

$$k = 0.002289(900)^{0.2068}$$

$$= 0.00934$$

Then,

$$\frac{\tau_o}{2q} = \frac{0.00934}{R_\theta^{\,0.2068}}$$

[redacted]

The formula for the turbulent portion of the boundary layer then becomes, for $H = 1.5$,

$$\left(\frac{\theta}{c}\right)_{\frac{s}{c}=1} = \frac{1}{(0.954)^{3.5}}\left[\frac{0.01126}{\left(2.675 \times 10^{6}\right)^{0.2068}}\int_{0.48}^{1}\left(\frac{U}{U_o}\right)^{4.017} d\frac{s}{c} + (0.0002901)^{1.207}(1.144)^{4.222}\right]^{1/1.207}$$

The value of $\int_{0.48}^{1}\left(\frac{U}{U_o}\right)^{4.017} d\frac{s}{c}$ is equal to 0.676

In figure 5 are shown curves of $\left(\frac{U}{U_o}\right)^{8.184}$ and $\left(\frac{U}{U_o}\right)^{4.017}$ that apply to the laminar and turbulent portions of the boundary layer, respectively. Then $\left(\frac{\theta}{c}\right)_{\frac{s}{c}=1} = 0.002005$ for one surface and

$$c_{d_o} = 4 \times 0.002005 \times 0.954^{3.25} = 0.0069$$

CONCLUSIONS

1. If the skin-friction coefficient is expressed as a power function of the Reynolds number based on the boundary-layer momentum thickness and the boundary-layer shape is assumed to be constant, it is possible to integrate von Kármán's boundary-layer momentum equation directly.

2. The integrated boundary-layer momentum equation permits the boundary-layer momentum thickness at the airfoil trailing edge to be rapidly determined for estimation of airfoil profile-drag coefficients.

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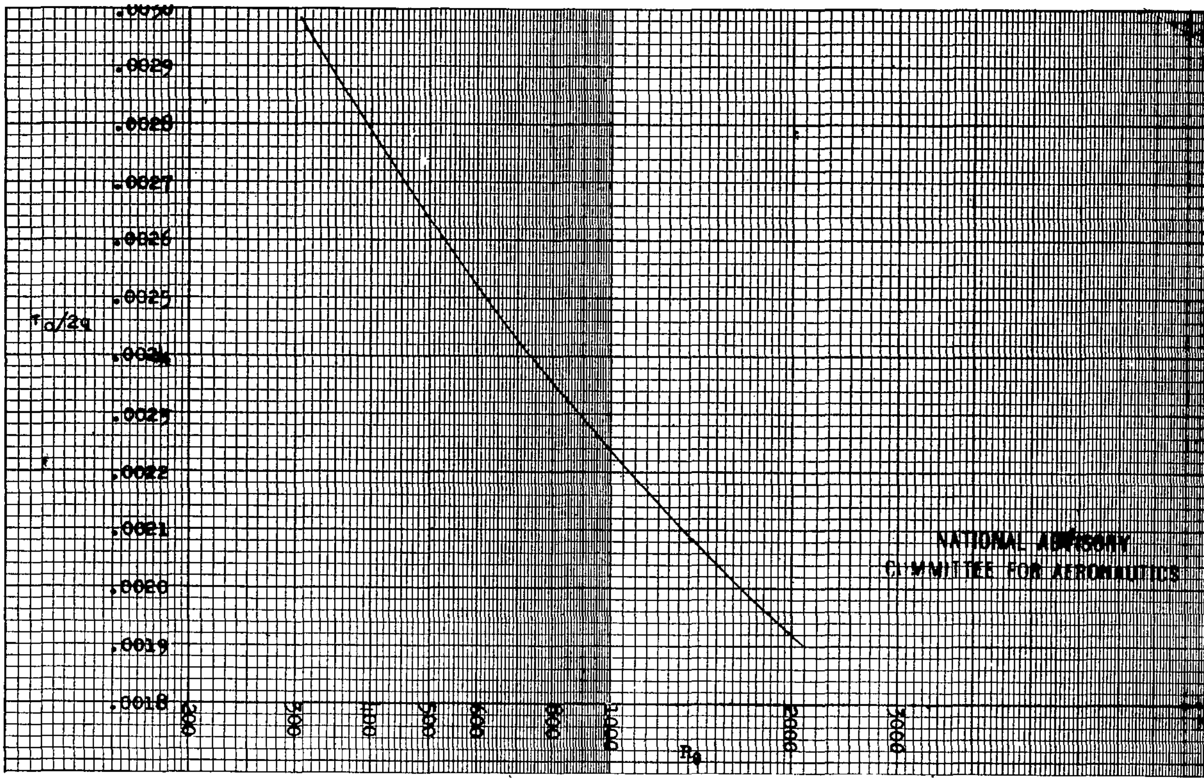

Figure 1.- Skin-friction coefficient $\tau_o/2q$ as a function of R_θ obtained by equation (2).

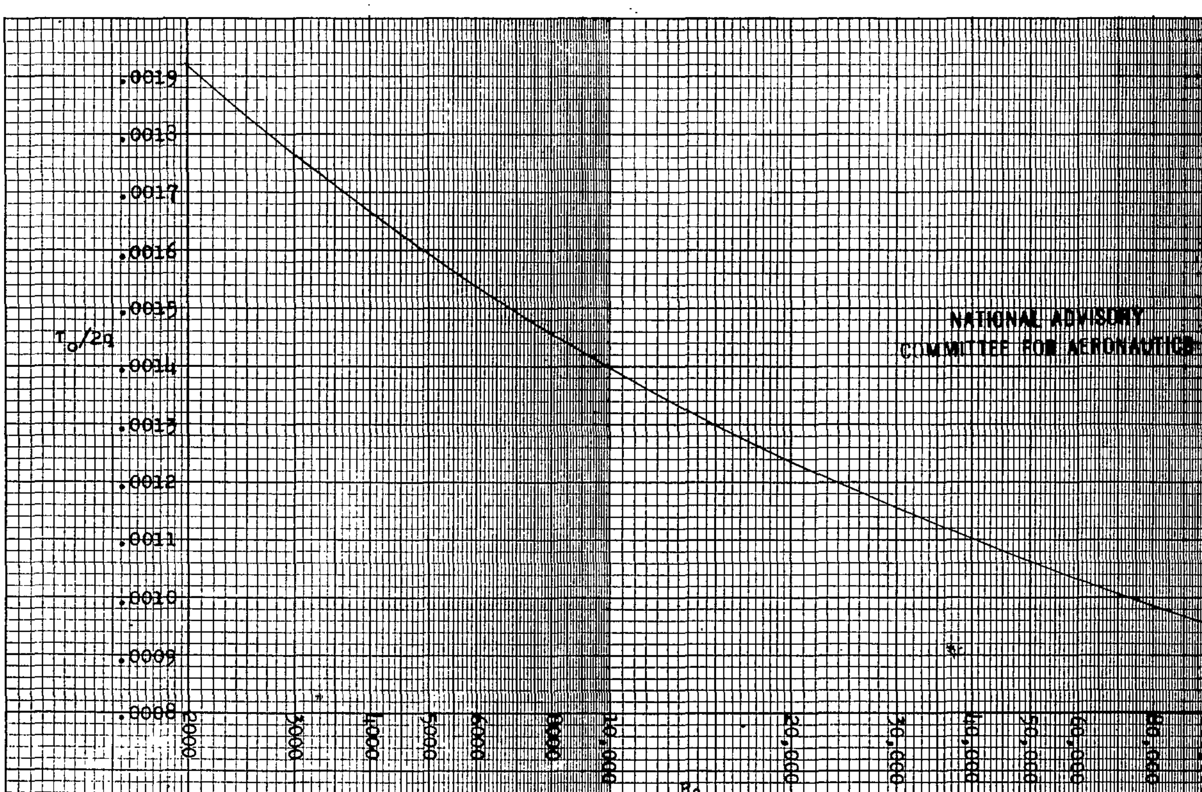

Figure 1.- Concluded.

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[redacted]

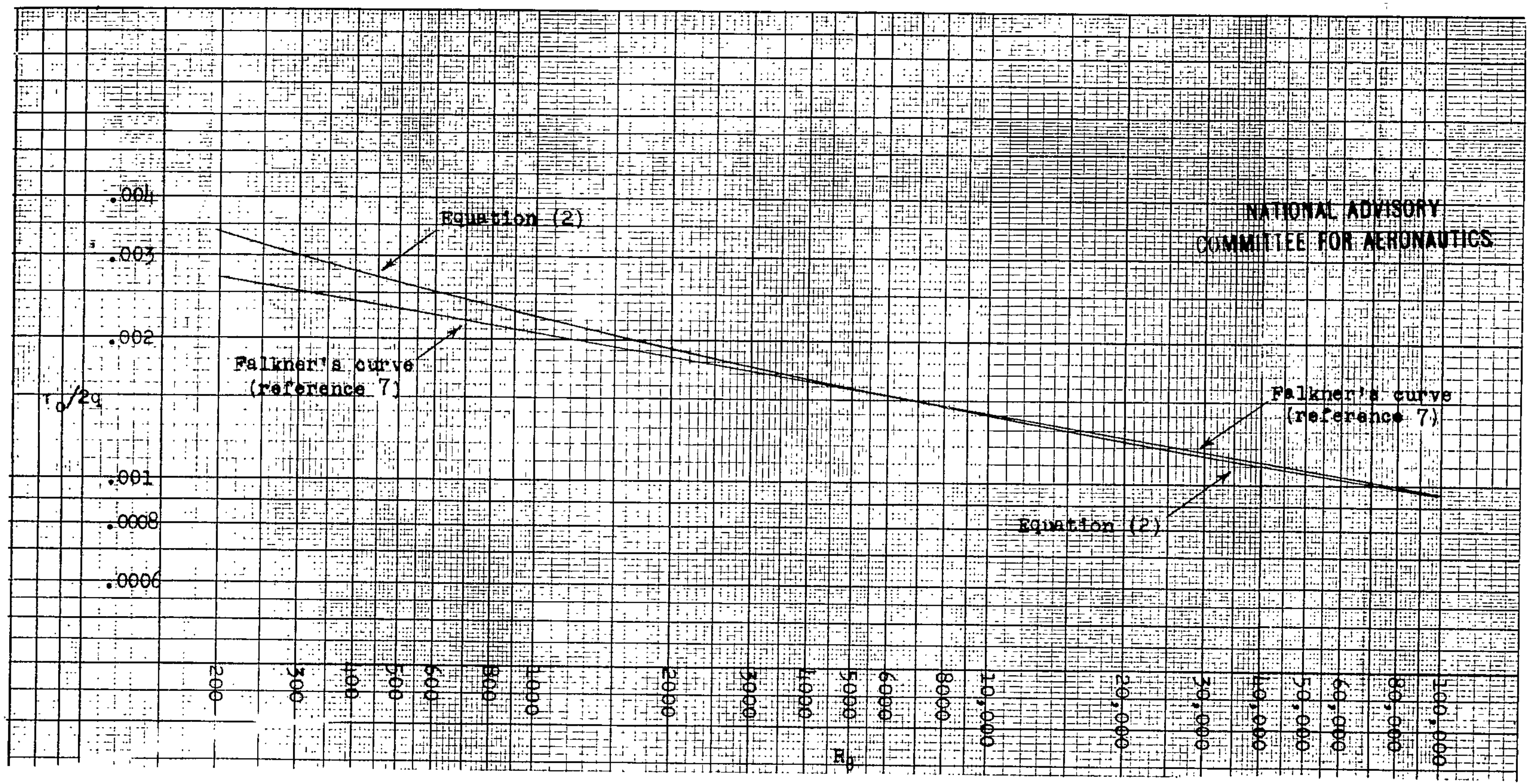

Figure 2.- Plot of Falkner's equation and equation (2) against R_θ.

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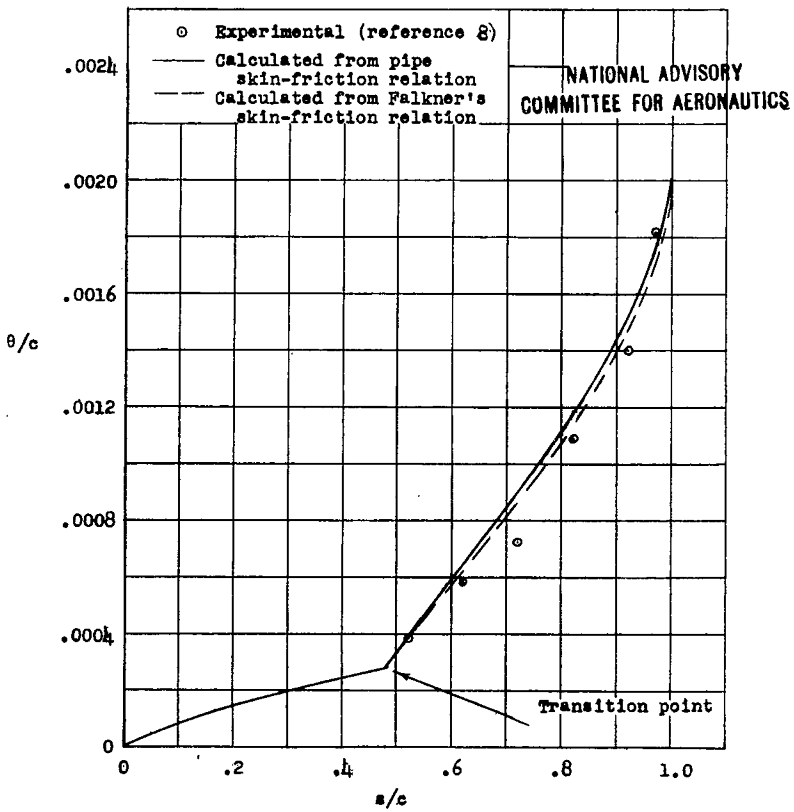
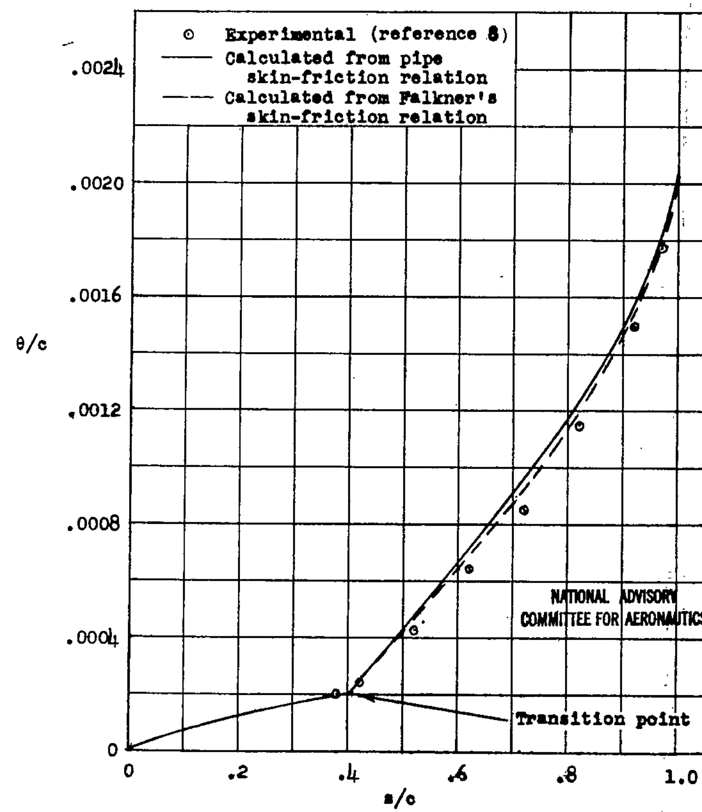


(a) R, 2.675 × 10^6.

Figure 3.- Experimental and calculated momentum thickness on the NACA 0012 airfoil.



(b) R, 3.780 × 10^6.

Figure 3.- Continued.

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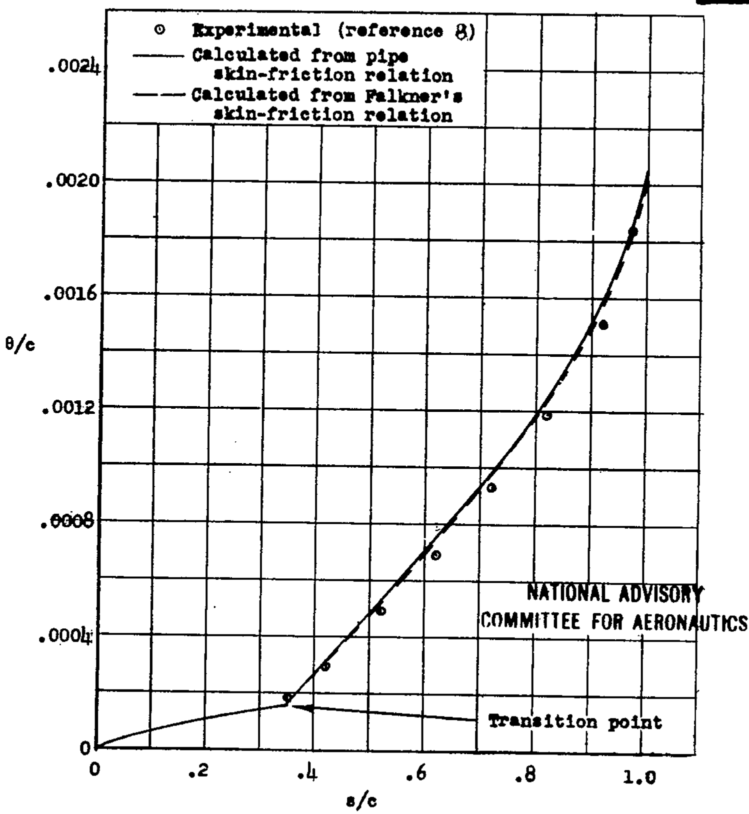

(c) R, 5.350×10^6.

Figure 3.- Continued.

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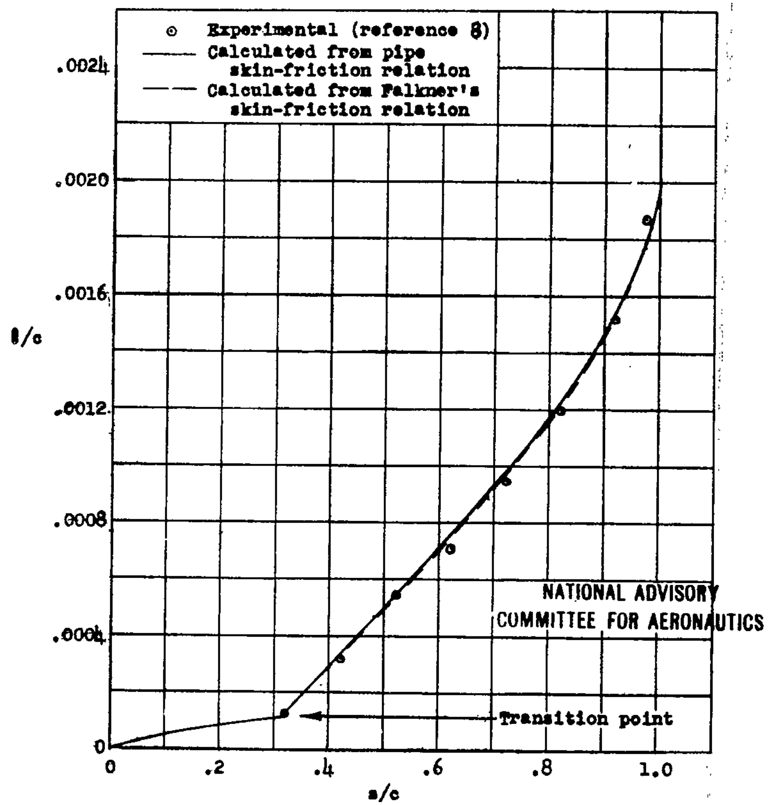

(d) R, 7.560×10^6.

Figure 3.- Concluded.

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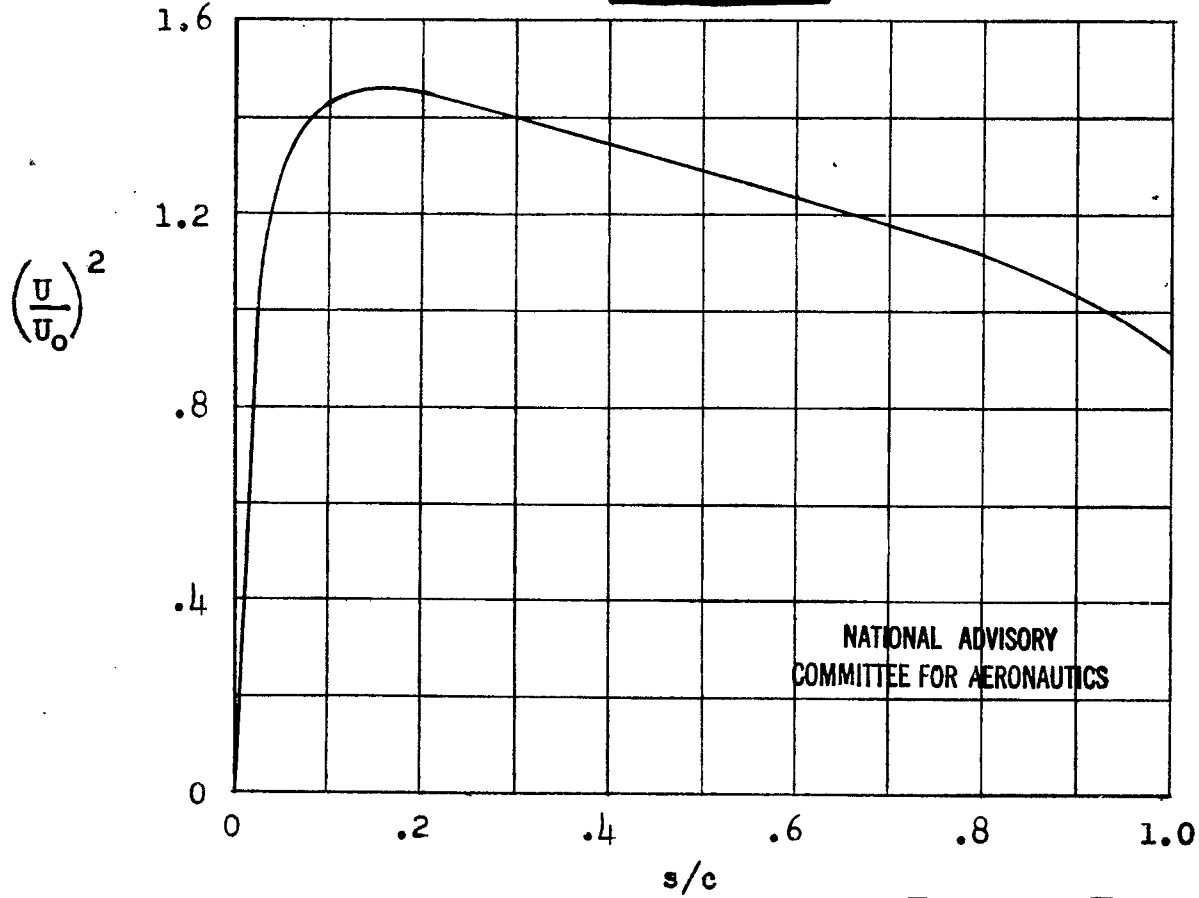

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Figure 4.- Velocity distribution for NACA 0012 airfoil at $\alpha = 0^o$.
(Taken from reference 8.)

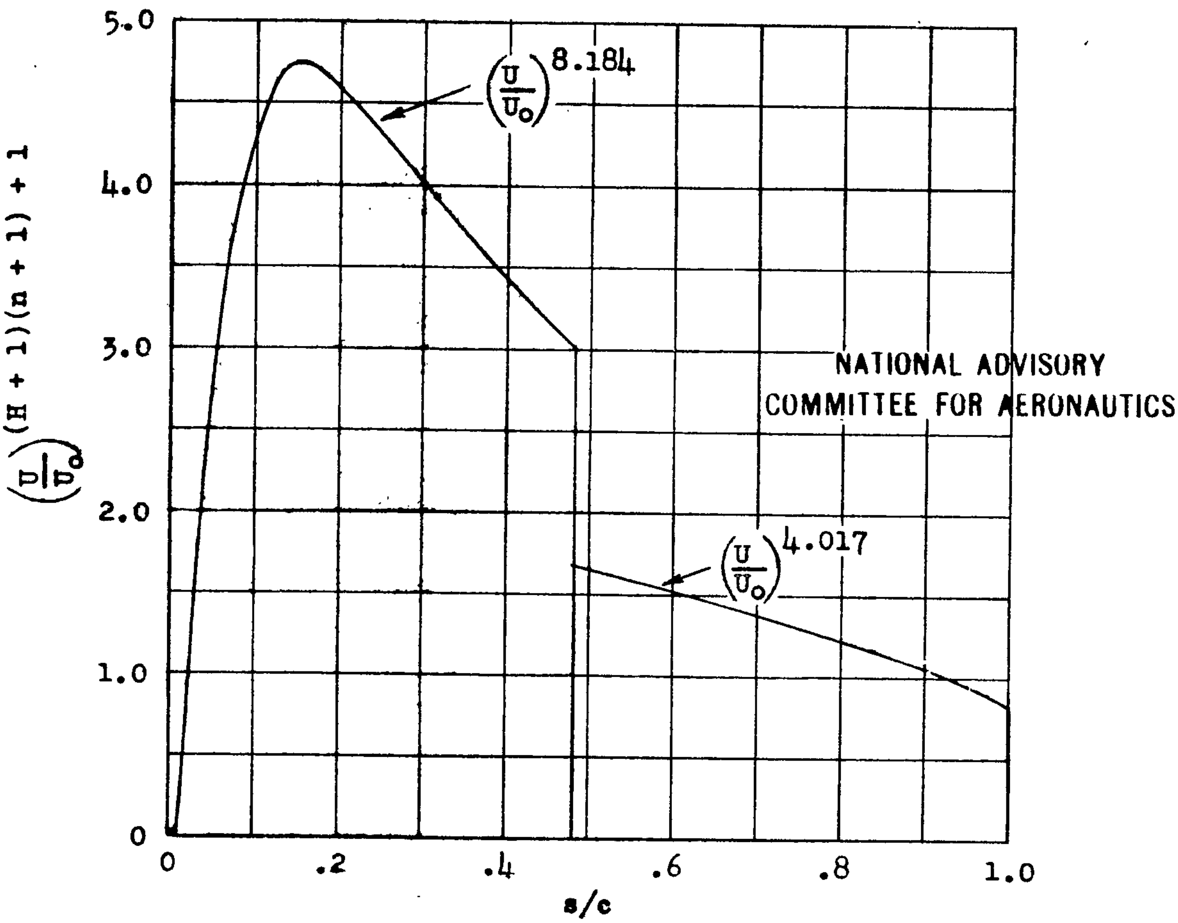

Figure 5.- Curves of $\left(\frac{U}{U_o}\right)^{(H + 1)(n + 1) + 1}$ for the NACA 0012 airfoil at $\alpha = 0^{\circ}$.

[redacted]